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Double diffractive ρ-production in $\gamma^*\gamma^*$ collisions

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Abstract. We present a first estimate of the cross-section for the exclusive process $\gamma^*_{\mathrm{L}}(Q_1^2)\gamma^*_{\mathrm{L}}(Q_2^2) \to \rho^0_{\mathrm{L}}\rho^0_{\mathrm{L}}$, which will be studied in the future high energy e^+e^--linear collider. As a first step, we calculate the Born order approximation of the amplitude for longitudinally polarized virtual photons and mesons, in the kinematical region $s \gg -t, Q_1^2, Q_2^2$. This process is completely calculable in the hard region $Q_1^2, Q_2^2 \gg \Lambda^2_{\mathrm{QCD}}$. We perform most of the steps in an analytical way. The resulting cross-section turns out to be large enough for this process to be measurable with foreseen luminosity and energy, for Q_1^2 and Q_2^2 in the range of a few GeV^2.

1 Introduction

The next generation of e^+e^--colliders will offer a possibility of clean testing of QCD dynamics. By selecting events in which two vector mesons are produced with large rapidity gap, through scattering of two highly virtual photons, one is getting access to the kinematical regime in which the perturbative approach is justified. If additionally one selects the events with comparable photon virtualities, the perturbative Regge dynamics of QCD of the BFKL [1] type should dominate with respect to the conventional partonic evolution of DGLAP [2] type. Apart from the study of the total cross-section which has been proposed as a test of BFKL dynamics [3, 4], one can achieve similar goals by studying diffractive reactions. From this point of view the production of two J/Ψ-mesons was studied in [5]. In this case the hard scale is supplied by the mass of the heavy quark. In the present paper we propose to study the electroproduction of two ρ-mesons in the $\gamma^*\gamma^*$ collisions. In this case the virtualities of the scattered photons play the role of the hard scales. As a first step in this direction we shall consider this process with longitudinally polarized photons and ρ-mesons,

$$\gamma^*_{\mathrm{L}}(q_1)\gamma^*_{\mathrm{L}}(q_2) \to \rho^0_{\mathrm{L}}(k_1)\rho^0_{\mathrm{L}}(k_2), \tag{1.1}$$

for arbitrary values of $t = (q_1 - k_1)^2$, with $s \gg -t$. The choice of longitudinal polarizations of both the scattered photons and produced vector mesons is dictated by the fact that this configuration of the lowest twist-2 gives the dominant contribution in the powers of the hard scale Q^2, when $Q_1^2 \sim Q_2^2 \sim Q^2$. The measurable cross-section is related to the amplitude of this process through the usual photon flux factors :

$$\frac{\mathrm{d}\sigma(e^+e^- \to e^+e^-\rho\rho)}{\mathrm{d}y_1\mathrm{d}y_2\mathrm{d}Q_1^2\mathrm{d}Q_2^2} \tag{1.2}$$
$$= \frac{1}{Q_1^2Q_2^2}\frac{\alpha}{2\pi}P_{\gamma/e}(y_1)P_{\gamma/e}(y_2)\sigma(\gamma^*\gamma^* \to \rho\rho),$$

where y_i are the longitudinal momentum fractions of the bremsstrahlung photons with respect to the respective leptons and with $P_{\gamma/e}(y) = 2(1-y)/y$ for longitudinally polarized photons. These double tagged events allow one to access this hard cross-section [6]. According to our best knowledge this process has not been discussed up to now. At lower energy, some experimental data exist for Q_2^2 small [7] and these may be analyzed [8] in terms of generalized distribution amplitudes [9] or transition distribution amplitudes [10].

In this paper we calculate the scattering amplitude of the process (1.1) in the Born approximation. In this way we get an estimate of the cross-section and prove the feasibility of a dedicated experiment. Partial results have been presented in [11]. In the near future, we intend to extend this study by taking into account BFKL evolution and transverse photon polarizations.

2 Kinematics

The process (1.1) at high energies can be visualized as in Fig. 1. The physical picture of this impact representation is due to the presence of different time scales: a $q\bar{q}$

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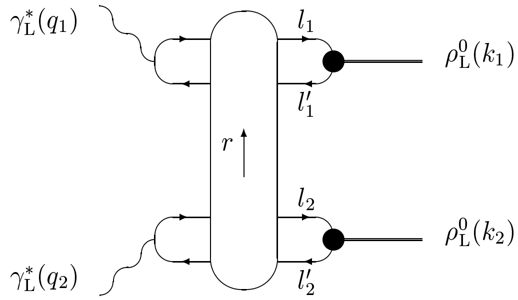

Fig. 1. Amplitude for the process $\gamma^*_{\rm L}\gamma^*_{\rm L} \to \rho^0_{\rm L}\rho^0_{\rm L}$ in the impact representation. The blobs denote the vector meson distribution amplitudes

dipole is formed from the virtual photon, then interacts by exchanging t-channel gluons before recombining in a meson. Figure 1 also explains the kinematics of the process (1.1). We parametrize the incoming photon momenta by introducing two light-like Sudakov vectors q'_1 and q'_2 related to the incoming particles, which satisfy $2q'_1 \cdot q'_2 = s \equiv 2q_1 \cdot q_2(1 - Q_1^2 Q_2^2/s^2)$. The usual $s_{\gamma^*\gamma^*}$ is related to the auxiliary useful variable s by $s_{\gamma^*\gamma^*} = s - Q_1^2 - Q_2^2 + Q_1^2 Q_2^2/s$. In this basis, the incoming photon momenta read

$$q_1 = q'_1 - \frac{Q_1^2}{s} q'_2,$$
$$q_2 = q'_2 - \frac{Q_2^2}{s} q'_1. \tag{2.1}$$

Their polarization vectors are

$$\epsilon^{\rm L(1)}_\mu = \frac{q_{1\mu}}{Q_1} + \frac{2Q_1}{s} q'_{2\mu},$$
$$\epsilon^{\rm L(2)}_\mu = \frac{q_{2\mu}}{Q_2} + \frac{2Q_2}{s} q'_{1\mu}, \tag{2.2}$$

which is obtained after imposing the conditions $\epsilon^2_{{\rm L}(i)} = 1$ and $q_i \cdot \epsilon_{{\rm L}(i)} = 0$. Because of electromagnetic gauge invariance, polarization vectors (2.2) can be effectively replaced by the second terms on the RHS of these equations. The momentum transfer in the t-channel is $r = k_1 - q_1$.

We label the momentum of the quarks and antiquarks entering the meson wave functions as l_1 and l'_1 for the upper part of the diagram and l_2 and l'_2 for the lower part (see Fig. 1).

In the basis (2.1), the vector meson momenta can be expanded in the form

$$k_1 = \alpha(k_1) q'_1 + \frac{\underline{r}^2}{\alpha s} q'_2 + r_\perp,$$
$$k_2 = \beta(k_1) q'_2 + \frac{\underline{r}^2}{\beta s} q'_1 - r_\perp. \tag{2.3}$$

Note that our convention is such that for any transverse vector $v_\perp$ in Minkowski space, $\underline{v}$ denotes its euclidean form. In the following, we will treat the ρ-meson as being massless. α and β are very close to unity, and read

$$\alpha(k_1) = \frac{1}{2}\left(1 - \frac{Q_2^2}{s}\right)$$
$$\times \left(1 + \sqrt{1 - 4\frac{\underline{r}^2}{s} \frac{1}{\left(1 - \frac{Q_1^2}{s}\right)\left(1 - \frac{Q_2^2}{s}\right)}}\right),$$
$$\beta(k_2) = \frac{1}{2}\left(1 - \frac{Q_1^2}{s}\right) \tag{2.4}$$
$$\times \left(1 + \sqrt{1 - 4\frac{\underline{r}^2}{s} \frac{1}{\left(1 - \frac{Q_1^2}{s}\right)\left(1 - \frac{Q_2^2}{s}\right)}}\right).$$

In this decomposition, it is straightforward to relate $t = r^2$ with $\underline{r}^2 = -r_\perp^2$. The corresponding relation is

$$t = -\frac{Q_1^2 Q_2^2}{s} - \frac{2\underline{r}^2}{1 + \sqrt{1 - \frac{4\underline{r}^2}{s\left(1 - \frac{Q_1^2}{s}\right)\left(1 - \frac{Q_2^2}{s}\right)}}} \times \left(\frac{1}{1 - \frac{Q_1^2}{s}} + \frac{1}{1 - \frac{Q_2^2}{s}} - 1\right), \tag{2.5}$$

or equivalently,

$$\underline{r}^2 = -\left(t + \frac{Q_1^2 Q_2^2}{s}\right) \frac{\left(1 - \frac{Q_1^2}{s}\right)\left(1 - \frac{Q_2^2}{s}\right)}{\left(1 - \frac{Q_1^2 Q_2^2}{s^2}\right)} \times \left(1 + \frac{t + \frac{Q_1^2 Q_2^2}{s}}{s\left(1 - \frac{Q_1^2 Q_2^2}{s^2}\right)}\right). \tag{2.6}$$

From (2.5) the threshold for $|t|$ is given by $|t|_{\min} = Q_1^2 Q_2^2/s$, corresponding to $r_\perp = 0$. In the kinematical range we are interested in, the relation (2.6) can be approximated as $\underline{r}^2 = -t$, in accordance with the usual result that r can be considered as purely transverse in the Regge limit.

We write the Sudakov decomposition of the quarks entering the ρ-mesons as

$$l_1 = z_1 q'_1 + l_{\perp 1} + z_1 r_\perp - \frac{(l_{\perp 1} + z_1 r_\perp)^2}{z_1 s} q'_2,$$
$$l'_1 = \bar{z}_1 q'_1 - l_{\perp 1} + \bar{z}_1 r_\perp - \frac{(-l_{\perp 1} + \bar{z}_1 r_\perp)^2}{\bar{z}_1 s} q'_2,$$
$$l_2 = z_2 q'_2 + l_{\perp 2} - z_2 r_\perp - \frac{(l_{\perp 2} - z_2 r_\perp)^2}{z_2 s} q'_1,$$
$$l'_2 = \bar{z}_2 q'_2 - l_{\perp 2} - \bar{z}_2 r_\perp - \frac{(-l_{\perp 2} - \bar{z}_2 r_\perp)^2}{\bar{z}_2 s} q'_1, \tag{2.7}$$

where we have explicitly separated the transverse momenta $l_{\perp 1}$ ($l_{\perp 2}$) of quark and antiquark forming the ρ_{L}-mesons with respect to the momentum k_1 (k_2). In the following we shall apply the collinear approximation which consists in putting the relative momenta $l_{i\perp}$ in (2.7) to zero at each $q\bar{q}\rho$-meson vertex. The decomposition (2.7) is more easily understood in a slightly different Sudakov basis, which up to term linear in $r_\perp$ is obtained by the substitutions $q_1' \to q_1' + r_\perp$ and $q_2' \to q_2' - r_\perp$; see (2.3) with $\alpha(k_1)$ and $\beta(k_2)$ close to unity. In this new basis the ρ-mesons have no transverse momenta and quarks transverse momenta are only relative.

3 Impact representation

The impact representation of the scattering amplitude for the process (1.1) has the form (see Fig. 2)

$$\begin{aligned}\mathcal{M} = \mathrm{i}s \int & \frac{\mathrm{d}^2\,\underline{k}}{(2\pi)^4 \underline{k}^2\,(\underline{r}-\underline{k})^2} \\ & \times \mathcal{J}^{\gamma^*_{\mathrm{L}}(q_1)\to\rho^0_{\mathrm{L}}(k_1)}(\underline{k},\underline{r}-\underline{k}) \\ & \times \mathcal{J}^{\gamma^*_{\mathrm{L}}(q_2)\to\rho^0_{\mathrm{L}}(k_2)}(-\underline{k},-\underline{r}+\underline{k}), \qquad (3.1)\end{aligned}$$

where $\mathcal{J}^{\gamma^*_{\mathrm{L}}(q_1)\to\rho^0_{\mathrm{L}}(k_1)}(\underline{k},\underline{r}-\underline{k})$ ($\mathcal{J}^{\gamma^*_{\mathrm{L}}(q_2)\to\rho^0_{\mathrm{L}}(k_2)}(\underline{k},\underline{r}-\underline{k})$) are the impact factors corresponding to the transition of $\gamma^*_{\mathrm{L}}(q_1)\to\rho^0_{\mathrm{L}}(k_1)$ ($\gamma^*_{\mathrm{L}}(q_2)\to\rho^0_{\mathrm{L}}(k_2)$) via the t-channel exchange of two gluons. The impact factors are the s-channel discontinuities of the corresponding S-matrices describing the $\gamma^*_{\mathrm{L}} g \to \rho^0_{\mathrm{L}} g$ processes projected on the longitudinal (nonsense) polarizations of the virtual gluons in the t-channel

$$\begin{aligned}&\mathcal{J}^{\gamma^*_{\mathrm{L}}(q_1)\to\rho^0_{\mathrm{L}}(k_1)}(\underline{k},\underline{r}-\underline{k}) \\ &= \int \frac{\mathrm{d}\beta}{s} S_{\mu\nu}^{\gamma^*_{\mathrm{L}}(q_1)\to\rho^0_{\mathrm{L}}(k_1)} q_2'^{\,\mu} q_2'^{\,\nu}, \qquad (3.2) \\ &\mathcal{J}^{\gamma^*_{\mathrm{L}}(q_2)\to\rho^0_{\mathrm{L}}(k_2)}(-\underline{k},-\underline{r}+\underline{k}) \\ &= \int \frac{\mathrm{d}\alpha}{s} S_{\mu\nu}^{\gamma^*_{\mathrm{L}}(q_2)\to\rho^0_{\mathrm{L}}(k_2)} q_1'^{\,\mu} q_1'^{\,\nu}.\end{aligned}$$

Here the integration variables α and β are defined by the Sudakov decomposition of the gluonic momentum k

$$k = \alpha q_1' + \beta q_2' + k_\perp. \qquad (3.3)$$

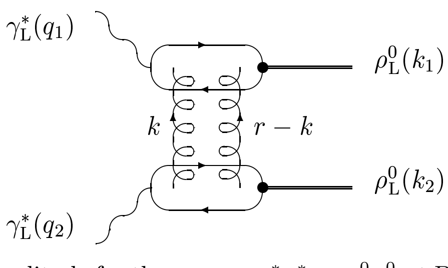

Fig. 2. Amplitude for the process $\gamma^*_{\mathrm{L}}\gamma^*_{\mathrm{L}} \to \rho^0_{\mathrm{L}}\rho^0_{\mathrm{L}}$ at Born order. The t-channel gluons are attached to the quark lines in all possible ways

The amplitude (3.1) depends linearly on s, since these impact factors are s-independent. Calculations of the impact factors in the Born approximation are standard [12].[1] They read

$$\begin{aligned}&\mathcal{J}^{\gamma^*_{\mathrm{L}}(q_1)\to\rho_{\mathrm{L}}(k_1)}(\underline{k},\underline{r}-\underline{k}) \\ &= \int_0^1 \mathrm{d}z_1 z_1 \bar{z}_1 \qquad (3.4) \\ &\quad \times \phi(z_1) 8\pi^2 \alpha_{\mathrm{s}} \frac{e}{\sqrt{2}} \frac{\delta^{ab}}{2N_c} Q_1 f_\rho \alpha(k_1) P_{\mathrm{P}}(z_1,\underline{k},\underline{r},\mu_1)\end{aligned}$$

and

$$\begin{aligned}&\mathcal{J}^{\gamma^*_{\mathrm{L}}(q_2)\to\rho^0_{\mathrm{L}}(k_2)}(-\underline{k},-\underline{r}+\underline{k}) \\ &= \int_0^1 \mathrm{d}z_2 z_2 \bar{z}_2 \qquad (3.5) \\ &\quad \times \phi(z_2) 8\pi^2 \alpha_{\mathrm{s}} \frac{e}{\sqrt{2}} \frac{\delta^{ab}}{2N_c} Q_2 f_\rho \beta(k_2) P_{\mathrm{P}}(z_2,\underline{k},\underline{r},\mu_2),\end{aligned}$$

where

$$\begin{aligned}P_{\mathrm{P}}(z_1,\underline{k},\underline{r},\mu_1) = & \frac{1}{z_1^2\underline{r}^2+\mu_1^2} + \frac{1}{\bar{z}_1^2\underline{r}^2+\mu_1^2} \qquad (3.6) \\ & - \frac{1}{(z_1\underline{r}-\underline{k})^2+\mu_1^2} - \frac{1}{(\bar{z}_1\underline{r}-\underline{k})^2+\mu_1^2}\end{aligned}$$

is proportional to the impact factor of quark pair production from a longitudinally polarized photon, with two t-channel exchanged gluons. In formula (3.6) the collinear approximation $l_\perp = 0$ has been made.

In this expression $\mu_1^2 = Q_1^2 z_1 \bar{z}_1 + m^2$ and $\mu_2^2 = Q_2^2 z_2 \bar{z}_2 + m^2$, where m is the quark mass. The limit $m \to 0$ is regular and we will restrict ourselves to the light quark case, taking thus $m = 0$. Note that (3.5) can be obtained from (3.4) from the combination of two substitutions, firstly $(z_1, Q_1) \to (z_2, Q_2)$ and secondly $(\underline{k},\underline{r}) \to (\underline{k}-\underline{r}, -\underline{r})$. This second substitution is easy to understand since for the upper (lower) blob, the total t-channel outgoing momentum is $r_\perp$ ($-r_\perp$), the outgoing gluon carries momentum k ($k-r$), and the incoming gluon carries momentum $k-r$ (k). This substitution effectively corresponds to exchanging the third and the fourth term in (3.6), and leaves this impact factor invariant, due to its symmetry under $z \to \bar{z}$, which is reminiscent of the pomeron structure of the t-channel state (namely, the impact factor is even under C conjugation). Thus, only the first substitution is necessary to obtain the upper blob from the lower blob.

In the formulae (3.4) and (3.5), ϕ is the distribution amplitude of the produced longitudinally polarized ρ-mesons. For the case with quark q of one flavor it is defined (see,

[1] Recently the forward impact factor of $\gamma^*_{\mathrm{L}}(Q^2) \to \rho^0_{\mathrm{L}}$ transition was calculated at the next-to-leading order accuracy in [13].

e.g. [14]) by the matrix element of the non-local, gauge invariant correlator of quark fields on the light-cone,

$$\langle 0|\bar{q}(x)\gamma^\mu q(-x)|\rho_{\mathrm{L}}(p)\rangle = f_\rho p^\mu \int_0^1 \mathrm{d}z \mathrm{e}^{\mathrm{i}(2z-1)(px)}\phi(z), \tag{3.7}$$

where the coupling constant is $f_\rho = 216\,\mathrm{MeV}$ and where the gauge links are omitted to simplify the notation. The amplitudes for the production of the ρ^0 are obtained by noting that $|\rho^0\rangle = 1/\sqrt{2}(|\bar{u}u\rangle - |\bar{d}d\rangle)$.

The structures in the formulae (3.4) and (3.5) arise due to the collinear approximation. Indeed, one can neglect the $l_\perp$-dependences in the propagators when computing the impact factors. Thus the only dependence with respect to this transverse momentum of the quark, in the Sudakov basis of the ρ-meson, is inside the wave function of these ρ-mesons. When integrating over the phase space of the quarks and antiquarks, this results in integrating the meson wave functions up to the factorization scale (which is of the order of the photon virtualities). We neglect here any evolution with respect to this scale. Such an integrated wave function is by definition the distribution amplitude of the ρ-mesons. For simplicity, we use the asymptotic distribution amplitude

$$\phi(z) = 6z(1-z). \tag{3.8}$$

The only remaining parts in the quark phase space integrations are the integration with respect to the quark longitudinal fractions of the meson momenta, z_1 and z_2.

Combining (3.1), (3.4), (3.5) and (3.6), the amplitude can be expressed as

$$\mathcal{M} = \mathrm{i}s2\pi \frac{N_c^2-1}{N_c^2}\alpha_{\mathrm{s}}^2\alpha_{\mathrm{em}}\alpha(k_1)\beta(k_2)f_\rho^2 Q_1 Q_2 \times \int_0^1 \mathrm{d}z_1\mathrm{d}z_2 z_1\bar{z}_1\phi(z_1)z_2\bar{z}_2\phi(z_2)\mathrm{M}(z_1,z_2), \tag{3.9}$$

with

$$\begin{aligned}\mathrm{M}(z_1,z_2) &= \int \frac{\mathrm{d}^2\underline{k}}{\underline{k}^2(\underline{r}-\underline{k})^2} \\ &\times \left[\frac{1}{z_1^2\underline{r}^2+\mu_1^2} + \frac{1}{\bar{z}_1^2\underline{r}^2+\mu_1^2} - \frac{1}{(z_1\underline{r}-\underline{k})^2+\mu_1^2} - \frac{1}{(\bar{z}_1\underline{r}-\underline{k})^2+\mu_1^2}\right] \\ &\times \left[\frac{1}{z_2^2\underline{r}^2+\mu_2^2} + \frac{1}{\bar{z}_2^2\underline{r}^2+\mu_2^2} - \frac{1}{(z_2\underline{r}-\underline{k})^2+\mu_2^2} - \frac{1}{(\bar{z}_2\underline{r}-\underline{k})^2+\mu_2^2}\right].\end{aligned} \tag{3.10}$$

In terms of this amplitude, the differential cross-section can be expressed in the large s limit (neglecting terms of order Q_i^2/s) as

$$\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{L}}^*\gamma_{\mathrm{L}}^*\to\rho_{\mathrm{L}}^0\rho_{\mathrm{L}}^0}}{\mathrm{d}t} = \frac{|\mathcal{M}|^2}{16\pi s^2}. \tag{3.11}$$

4 Cross section at Born order

4.1 Forward case

We begin with the simpler case $t = t_{\min}$ (i.e. $\underline{r} = 0$), where the final result for the function $M(z_1, z_2)$ (3.10) can be written in a rather simple form. The integral over $\underline{k}$ can readily be performed and gives

$$M(z_1,z_2) = \frac{4\pi}{z_1\bar{z}_1 z_2\bar{z}_2 Q_1^2 Q_2^2\,(z_1\bar{z}_1Q_1^2 - z_2\bar{z}_2Q_2^2)} \ln\frac{z_1\bar{z}_1Q_1^2}{z_2\bar{z}_2Q_2^2}. \tag{4.1}$$

The amplitude $\mathcal{M}$ given by (3.9) can then be computed analytically from (4.1) through double integration over z_1 and z_2. This is explained in Appendix A.1. It results in

$$\begin{aligned}\mathcal{M}_{t_{\min}} &= -\mathrm{i}s\frac{N_c^2-1}{N_c^2}\alpha_{\mathrm{s}}^2\alpha_{\mathrm{em}}\alpha(k_1)\beta(k_2)f_\rho^2\frac{9\pi^2}{2}\frac{1}{Q_1^2Q_2^2} \\ &\times\Bigg[6\left(R+\frac{1}{R}\right)\ln^2 R + 12\left(R-\frac{1}{R}\right)\ln R \\ &+12\left(R+\frac{1}{R}\right) + \left(3R^2+2+\frac{3}{R^2}\right) \\ &\times\big(\ln(1-R)\ln^2 R - \ln(R+1)\ln^2 R \\ &-2\mathrm{Li}_2(-R)\ln R + 2\mathrm{Li}_2(R)\ln R \\ &+2\mathrm{Li}_3(-R) - 2\mathrm{Li}_3(R)\big)\Bigg],\end{aligned} \tag{4.2}$$

where $R = Q_1/Q_2$.

In the special case where $Q = Q_1 = Q_2$, it simplifies immediately to

$$\mathcal{M}_{t_{\min}}(Q_1=Q_2) \sim -\mathrm{i}s\frac{N_c^2-1}{N_c^2}\alpha_{\mathrm{s}}^2\alpha_{\mathrm{em}}\alpha(k_1)\beta(k_2)f_\rho^2\frac{9\pi^2}{2Q^4}\left(24-28\zeta(3)\right). \tag{4.3}$$

The peculiar limits $R \gg 1$ and $R \ll 1$ are of special physical interest, since they correspond to the kinematics typical for deep inelastic scattering on a photon target described through collinear approximation, i.e. the usual parton model. At moderate values of s, apart from diagrams with gluon exchange in the t-channel considered here, one should also take into account diagrams with quark exchange. We do not consider them here since we restrict ourselves to the asymptotical region of large s.

In the limit $R \gg 1$, the amplitude simplifies into

$$\mathcal{M}_{t_{\min}} \tag{4.4}$$
$$\sim \mathrm{i}s\frac{N_c^2-1}{N_c^2}\alpha_s^2\alpha_{\mathrm{em}}\alpha(k_1)\beta(k_2)f_\rho^2\frac{96\pi^2}{Q_1^2Q_2^2}\left(\frac{\ln R}{R}-\frac{1}{6R}\right).$$

This result can be obtained directly by imposing from the very beginning the $k_\perp$ ordering typical for the parton model. This is shown explicitly in Appendix A.2.

4.2 Non-forward case

In this section we will compute the amplitude (3.1); we have been able to perform analytically the $k_\perp$ integrals. It involves the evaluation of a box diagram with two distinct massive propagators and two massless propagators (denoted $I_{4m_am_b}$ below). We are not aware of any previous analytic calculation of such an integral. This gives us the possibility of studying various kinematical limits in the variables Q_1^2, Q_2^2, t.

$M(z_1,\, z_2)$ as defined in (3.10) is symmetric under $z_1 \leftrightarrow \bar{z}_1$ and under $z_2 \leftrightarrow \bar{z}_2$. Since on the other hand the distribution amplitude $\phi(z)$ is also symmetric under $z \leftrightarrow \bar{z}$, $M(z_1, z_2)$ can be modified by adding any antisymmetric term, since the integration over z_1 and z_2 will automatically select its symmetric part.

One thus writes

$$\mathcal{M} = \mathrm{i}s2\pi\frac{N_c^2-1}{N_c}\alpha_s^2\alpha_{\mathrm{em}}\alpha(k_1)\beta(k_2)f_\rho^2Q_1Q_2 \tag{4.5}$$
$$\times\int_0^1 \mathrm{d}z_1\mathrm{d}z_2 z_1\bar{z}_1\phi(z_1)z_2\bar{z}_2\phi(z_2)M_{\mathrm{asym.}}(z_1,z_2),$$

with

$$M_{\mathrm{asym.}}(z_1,z_2)$$
$$= \int\frac{\mathrm{d}^2\underline{k}}{\underline{k}^2(\underline{r}-\underline{k})^2}\left[\frac{1}{(z_1^2\underline{r}^2+\mu_1^2)\,(z_2^2\underline{r}^2+\mu_2^2)}\right.$$
$$-\frac{1}{(z_1^2\underline{r}^2+\mu_1^2)\left((z_2\underline{r}-\underline{k})^2+\mu_2^2\right)}$$
$$-\frac{1}{(z_2^2\underline{r}^2+\mu_2^2)\left((z_1\underline{r}-\underline{k})^2+\mu_1^2\right)}$$
$$\left.+\frac{1}{\left((z_1\underline{r}-\underline{k})^2+\mu_1^2\right)\left((z_2\underline{r}-\underline{k})^2+\mu_2^2\right)}\right]. \tag{4.6}$$

$M_{\mathrm{asym.}}(z_1, z_2)$ can be expressed in terms of three kinds of integrals, namely,

$$I_2 = \int\frac{\mathrm{d}^d\underline{k}}{\underline{k}^2(\underline{k}-\underline{p})^2}, \tag{4.7}$$

$$I_{3m} = \int\frac{\mathrm{d}^d\underline{k}}{\underline{k}^2(\underline{k}-\underline{p})^2\left((\underline{k}-\underline{a})^2+m^2\right)}, \tag{4.8}$$

and

$$I_{4m_am_b} = \int\frac{\mathrm{d}^d\underline{k}}{\underline{k}^2(\underline{k}-\underline{p})^2\left((\underline{k}-\underline{a})^2+m_a^2\right)\left((\underline{k}-\underline{b})^2+m_b^2\right)}, \tag{4.9}$$

where we use the dimensional regularization $d = 2 + 2\epsilon$. However, since we will effectively rely, for computing the various integrals involved, on a method which is applicable only for both UV and IR finite integrals, it is more efficient to rewrite $\mathcal{M}$, (3.9), in terms of another asymmetric two dimensional amplitude, of the form

$$\tilde{M}(z_1,z_2)$$
$$= -\left(\frac{1}{z_1^2\underline{r}^2+\mu_1^2}+\frac{1}{\bar{z}_1^2\underline{r}^2+\mu_2^2}\right)J_{3\mu_2}(z_2) - (1\leftrightarrow\ 2)$$
$$+J_{4\mu_1\mu_2}(z_1,z_2)+J_{4\mu_1\mu_2}(\bar{z}_1,\, z_2), \tag{4.10}$$

where

$$J_{3\mu}(a) = \int\frac{\mathrm{d}^2\underline{k}}{\underline{k}^2(\underline{k}-\underline{r})^2} \tag{4.11}$$
$$\times\left[\frac{1}{(\underline{k}-\underline{r}a)^2+\mu^2}-\frac{1}{a^2\underline{r}^2+\mu^2}+(a\leftrightarrow\underline{a})\right],$$

and

$$J_{4\mu_1\mu_2}(z_1,z_2)$$
$$= \int\frac{\mathrm{d}^2\underline{k}}{\underline{k}^2(\underline{k}-\underline{r})^2}$$
$$\times\left[\frac{1}{\left((\underline{k}-\underline{r}z_1)^2+\mu_1^2\right)\left((\underline{k}-\underline{r}z_2)^2+\mu_2^2\right)}\right.$$
$$\left.-\frac{1}{(z_1^2\underline{r}^2+\mu_1^2)(z_2^2\underline{r}^2+\mu_2^2)}+(z\leftrightarrow\bar{z})\right]. \tag{4.12}$$

$J_{3\mu}$ and $J_{4\mu_1\mu_2}$ are two dimensional integrals with respectively three propagators (one massive) and four propagators (two massive, with different masses). They are both IR and UV finite. Their computation by a brute force technique, using Feynman parametrization, seems untractable in such a form (specially for $J_{4\mu_1\mu_2}$). However, applying a trick inspired by conformal field theories, it is possible to compute these integrals. The basic idea is to perform special conformal inverse transformations, considered here in momentum space. Although these two integrals, because of the mass terms, are not conformal invariant, this is actually efficient after a suitable redefinition of the massive parameters. This is presented in the Appendix A.3.

To complete the evaluation of the amplitude $\mathcal{M}$, one needs to integrate over the quark momentum fractions in the ρ-mesons z_1 and z_2. In the general case, for arbitrary values of t, it seems not possible to perform the z_1 and z_2 integrations analytically. We thus do them numerically. In the course of these calculations, we observe the absence of an end-point singularity when $z_{1(2)} \to 0$ or $z_{1(2)} \to 1$, since P_{P} as defined in (3.6) diverges like $1/z, 1/(1-z)$ when $z \to 0, 1$. This leads to perfectly stable numerical integrations.

4.3 Results

We now use the previous formulae in order to get a prediction for the production rate of diffractive double ρ production. The running of α_{s} is a subleading effect with respect to our treatment. Anyway, we choose to replace α_{s}^2 in the various formulae presented above by $\alpha_{\mathrm{s}}(Q_1^2)\alpha_{\mathrm{s}}(Q_2^2)$ in order to fix the coupling, and use the three-loop running coupling $\alpha_{\mathrm{s}}(Q_1^2)$ and $\alpha_{\mathrm{s}}(Q_2^2)$ with $\Lambda_{\overline{MS}}^{(4)} = 305\,\mathrm{MeV}$ (see, e.g. [15]).

In Fig. 3, we display the differential cross-section $\mathrm{d}\sigma(\gamma_{\mathrm{L}}^*\gamma_{\mathrm{L}}^* \to \rho_{\mathrm{L}}^0\rho_{\mathrm{L}}^0)/\mathrm{d}t$ for vanishing transverse t-channel momentum, i.e. $t = t_{\min}$, as a function of the ratio Q_2^2/Q_1^2. The curves are labeled by the values of Q_1. The dots on the curves represent the values of the cross-section at the special point $Q_1 = Q_2$, which obviously correspond to the analytical formula (4.3). The cross-section dramatically decreases when Q_2^2/Q_1^2 increases at fixed Q_1. For comparison, we show for each value of Q_1 the asymptotical curve obtained by combining (4.2) with (3.11). The complete result quickly approaches its asymptotical curve. However, in view of the strong decrease of the differential cross-section with increasing Q_2^2/Q_1^2, the asymptotical result seems to be of little interest for estimating the data rates in the most favorable kinematics.

The t-dependence of the differential cross-section $\mathrm{d}\sigma/\mathrm{d}t$ is displayed in Fig. 4 for various values of $Q = Q_1 = Q_2$.

As anticipated, the cross-section is strongly peaked in the forward direction. This fact is less dangerous than for the real photon case since the virtual photon is not in the direction of the beam. However, the differential cross-section

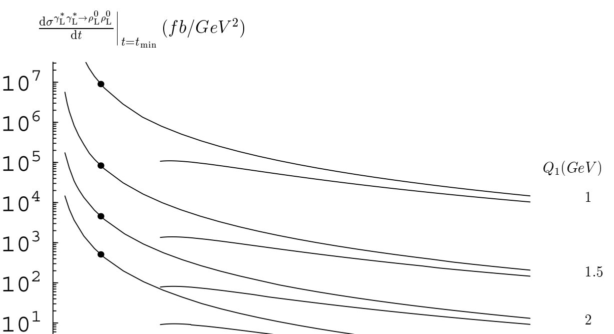

Fig. 3. Differential cross-section for the process $\gamma_{\mathrm{L}}^*\gamma_{\mathrm{L}}^* \to \rho_{\mathrm{L}}^0\rho_{\mathrm{L}}^0$ at Born order, at the threshold $t = t_{\min}$, as a function of Q_2^2/Q_1^2. The dots represents the value of the cross-section at the special point $Q_1 = Q_2$, as given by the analytical formula (4.3). The asymptotical curves are valid for large Q_2^2/Q_1^2, as predicted by the asymptotical form (4.4)

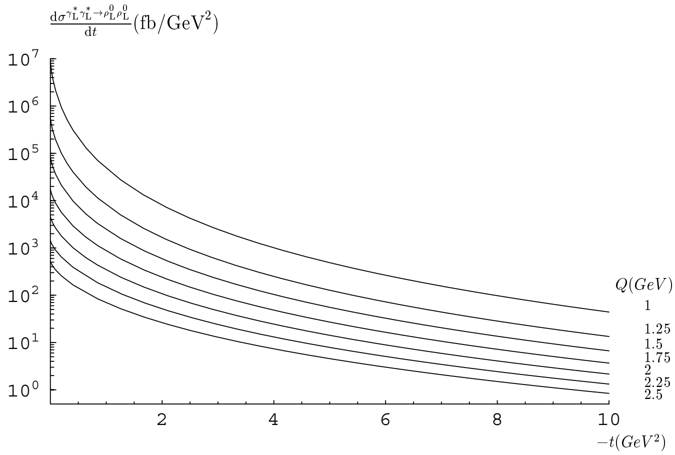

Fig. 4. Differential cross-section for the process $\gamma_{\mathrm{L}}^*\gamma_{\mathrm{L}}^* \to \rho_{\mathrm{L}}^0\rho_{\mathrm{L}}^0$ at Born order, as a function of t, for different values of $Q = Q_1 = Q_2$

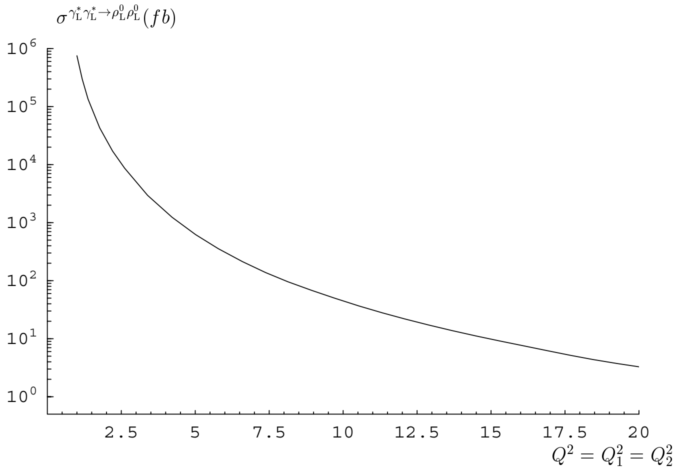

Fig. 5. The integrated cross-section for the process $\gamma_{\mathrm{L}}^*\gamma_{\mathrm{L}}^* \to \rho_{\mathrm{L}}^0\rho_{\mathrm{L}}^0$ at Born order as a function of $Q_1^2 = Q_2^2$

seems to be sufficient for the t-dependence to be measured up to a few GeV^2. The comparison of the curves on Fig. 3 for $t = t_{\min}$ with those on Fig. 4 leads to the conclusion that the phenomenological predictions obtained in the forward case will practically dictate the general trends of the integrated cross-sections.

Figure 5 shows the cross-section integrated over t as a function of $Q^2 = Q_1^2 = Q_2^2$. The magnitude of the cross-section seems to be sufficient for a detailed study to be performed at the linear collider presently under study. Note that we did not multiply by the virtual photon fluxes, which would amplify the dominance of smaller Q^2. However, the triggering efficiency often increases substantially with Q^2 [4]. At this level of calculation there is no s-dependence of the cross-section. It will appear after taking into account BFKL evolution.

5 Conclusion

This Born order study shows that the process $\gamma^*(Q_1^2)\gamma^*(Q_2^2) \to \rho_{\mathrm{L}}^0\rho_{\mathrm{L}}^0$ can be measured at foreseen e^+e^--

colliders for Q_1^2, Q_2^2 up to a few GeV^2. Indeed, a nominal integrated luminosity of $100\,\mathrm{fb}^{-1}$ should yield thousands of events per year, with $Q^2 \gtrsim 1\,\mathrm{GeV}^2$. This would open a new domain of investigation for diffractive processes in which practically all ingredients of the scattering amplitude are under control within a perturbative approach.

In the near future we expect to include the transversely polarized photon contribution, which should slightly enhance the non-forward amplitude (this amplitude obviously vanishes at $t = t_{\min}$). We also intend to incorporate the effect of BFKL evolution, since resummation effects are expected to give a net and visible enhancement of the cross-section.

Finally, let us note that the elusive odderon may also be looked for in $\gamma^*\gamma^*$ exclusive reactions [16], and that one may use the strategy developed in [17] to find it through its interference with pomeron exchange which gives rise to charge asymmetries in $\gamma^*\gamma^* \to \pi\pi\pi\pi$.[2]

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A Appendices

A.1 Computation of the amplitude at $t = t_{\min}$

In this appendix we will prove the result (4.2) for the production amplitude at $|t|_{\min} = Q_1^2 Q_2^2/s$, still neglecting the ρ mass. The amplitude (3.9) can be written as

$$\mathcal{M} = \mathrm{i}s\,\frac{N_c^2-1}{N_c^2}\,\alpha_{\mathrm{s}}^2\alpha_{\mathrm{em}}\alpha(k_1)\beta(k_2)f_\rho^2\,\frac{288\pi^2}{Q_1Q_2}K, \tag{A.1}$$

where

$$K = \int_0^1 \mathrm{d}z_1 \int_0^1 \mathrm{d}z_2 \int_0^\infty \mathrm{d}k^2\, \frac{z_1\bar{z}_1z_2\bar{z}_2}{(k^2+z_1\bar{z}_1Q_1^2)(k^2+z_2\bar{z}_2Q_2^2)}, \tag{A.2}$$

which reduces to

$$K = \int_0^1 \mathrm{d}z_1 \int_0^1 \mathrm{d}z_2\, \frac{z_1\bar{z}_1z_2\bar{z}_2}{z_1\bar{z}_1Q_1^2 - z_2\bar{z}_2Q_2^2}\ln\frac{z_1\bar{z}_1Q_1^2}{z_2\bar{z}_2Q_2^2}. \tag{A.3}$$

In the following, we denote $R = Q_1/Q_2$. We first reduce the computation of K to a one dimensional integral evaluation. Performing the change of variables $x_1 = 4z_1\bar{z}_1$ and $x_2 = 4z_2\bar{z}_2$, K reads

$$K = \frac{1}{4^2Q_1^2}\int_0^1 \frac{\mathrm{d}x_1}{\sqrt{1-x_1}}\,\frac{\mathrm{d}x_2x_2}{\sqrt{1-x_2}}\left(1-\frac{1}{1-R^2\,x_1/x_2}\right)$$
$$\times\ln\left(R^2\frac{x_1}{x_2}\right). \tag{A.4}$$

After replacing the variable x_1 by $x = x_1/x_2$, K reads

$$K = \frac{1}{4^2Q_1^2}\int_0^1 \frac{\mathrm{d}x_2x_2^2}{\sqrt{1-x_2}}\int_0^{1/x_2}\frac{\mathrm{d}x}{\sqrt{1-xx_2}}\left(1-\frac{1}{1-R^2x}\right)$$
$$\times\ln(R^2x). \tag{A.5}$$

One can now split the integration domain $x_2 \in [0,1] \times x \in [0,1/x_2]$ as $x \in [0,1] \times x_2 \in [0,1]$ and $x \in [1,\infty] \times x_2 \in [0,1/x]$. The integral corresponding to the second domain is identical to the first one after exchanging Q_1 and Q_2. Performing the integration over x_2 in the first domain, one gets for K

$$K = \frac{1}{128Q_1^2}$$
$$\times\int_0^1\frac{\mathrm{d}x}{x^{5/2}}\left[-6\sqrt{x}\,(1+x)\right.$$
$$\left.+(3+2x+3x^2)\left(2\,\ln(1+\sqrt{x})\ln(1-x)\right)\right]$$
$$\times\left(1-\frac{1}{1-R^2x}\right)\ln(R^2x) + (1\leftrightarrow 2). \tag{A.6}$$

This one dimensional integral, after the change of variable $t = \sqrt{x}$, can finally be reexpressed as

$$K = -\frac{1}{64Q_1Q_2}$$
$$\times\int_0^1\mathrm{d}t\left\{\frac{6R}{t^2}\left(-2t+\ln(1+t)-\ln(1-t)\right)\ln(R\,t)\right.$$
$$-6(1+R^2)\left(\frac{1}{1-Rt}-\frac{1}{1+R\,t}\right)\ln(Rt)$$
$$-\frac{6}{R}\ln(1+t)\ln(R\,t)+\frac{6}{R}\ln(1-t)\ln(Rt)$$
$$+R\left(3R^2+2+\frac{3}{R^2}\right)$$
$$\times\left(\frac{\ln(1+t)\ln(Rt)}{1-Rt}+\frac{\ln(1+t)\ln(Rt)}{1+R\,t}\right.$$
$$\left.\left.-\frac{\ln(1-t)\ln(Rt)}{1-R\,t}-\frac{\ln(1-t)\ln(Rt)}{1+Rt}\right)\right\}$$
$$+\left(R\leftrightarrow\frac{1}{R}\right). \tag{A.7}$$

The integrals arising from the four first lines of the previous expression, supplemented by the corresponding $R \to 1/R$ contribution, are easily computed by integration by parts through the logarithmic and polylogarithmic Li_2 functions.

[2] After this paper has been accepted, two works [21] improve our analysis by taking into account higher order effects.

Using the Landen relation for Li_2 (see Chapter 1 of [18]), this simplifies to

$$A = -\frac{1}{64Q_1Q_2}\left[6\left(R+\frac{1}{R}\right)\ln^2 R + 12\left(R-\frac{1}{R}\right)\ln R + 12\left(R+\frac{1}{R}\right)\right]. \qquad (A.8)$$

The four last lines of (A.7) (denoted B in the following) contain terms of the generic form

$$\int \frac{\ln(a+bx)\ln(c+ex)}{f+gx} g\mathrm{d}x, \qquad (A.9)$$

which can be reduced to the standard form (see Chapter 8 of [18])

$$\int \ln(1-y)\ln(1-cy)\frac{\mathrm{d}y}{y}. \qquad (A.10)$$

This last integral is evaluated in Chapter 8 of [18] for a restricted domain in c. An analytic continuation of such a result for the whole complex domain in c can also be performed [19]. Let us first define

$$\varphi(\alpha) = \arg(\mathrm{e}^{\mathrm{i}\alpha}) - \alpha \qquad (A.11)$$

for any *real* alpha. With this definition, $\frac{\phi(\alpha)}{2\pi}$ is a winding number which counts the number of turns one has to make around 0 in order to bring back α to a value inside the interval $]-\pi,\pi]$. Then the result for the integral (A.10) reads, for x real,

$$\begin{aligned}
&\int_0^x \ln(1-y)\ln(1-cy)\frac{\mathrm{d}y}{y} \\
&= \mathrm{Li}_3\left(\frac{1-cx}{1-x}\right) + \mathrm{Li}_3\left(\frac{1}{c}\right) + \mathrm{Li}_3(1) - \mathrm{Li}_3(1-c\,x) \\
&-\mathrm{Li}_3(1-x) - \mathrm{Li}_3\left(\frac{1-cx}{c(1-x)}\right) \\
&+\ln(1-x)\mathrm{Li}_2(1-cx) - \ln(1-cx)\mathrm{Li}_2(x) \\
&+(\ln(1-cx)-\ln(1-x))\,\mathrm{Li}_2\left(\frac{1}{c}\right) \\
&+\frac{\pi^2}{6}\ln(1-x) + \frac{1}{2}\ln c\ln^2(1-x) \\
&-\mathrm{i}\pi c_1\ln^2 c + \mathrm{i}2\pi c_1\ln c\ln(1-cx) \\
&-\mathrm{i}2\pi c_1\ln c\ln(1-x) - 4\pi^2 c_2c_3\ln c \\
&+\mathrm{i}2\pi(c_1-c_4-c_5)\ln(1-x)\ln(1-cx) \\
&-4\pi^2 c_2c_3\ln(1-x) + 4\pi^2 c_2c_3\ln(1-c\,x) \\
&-\pi(c_1-c_4-c_5)\mathrm{i}\ln^2(1-c\,x) \\
&+\mathrm{i}\pi(-c_1+c_5)\ln^2(1-x) + \mathrm{i}4\pi^3 c_2c_3, \qquad (A.12)
\end{aligned}$$

where

$$\begin{aligned}
c_1 &= \frac{1}{2\pi}\varphi\left(\arg(c-1) - \arg(c) - \arg(1-x)\right), \\
c_2 &= \frac{1}{2\pi}\varphi\left(-\arg(c)\right), \\
c_3 &= \frac{1}{2\pi}\varphi\left(-\arg(1-x)\right), \\
c_4 &= \frac{1}{2\pi}\varphi\left(\arg(c)+\arg(x)\right), \qquad (A.13)\\
c_5 &= \frac{1}{2\pi}\varphi\left(\arg(x)+\arg(c-1)-\arg(1-x)\right).
\end{aligned}$$

c_1, c_2, c_3 and c_4 take either the value 0 or +1, while c_5 can take the -1, 0 or 1 values. From this formula one can get an analytical form for B. The obtained result is very lengthy and contains a large bunch of ln, Li_2 and Li_3 of various rational functions of R. A first simplification occurs when using the Landen relation for Li_3:

$$\begin{aligned}
\mathrm{Li}_3(x) = \mathrm{Li}_3\left(\frac{1}{x}\right)
&-\frac{\pi^2}{6}\ln(-x) - \frac{1}{6}\ln^3(-x) \qquad (A.14)\\
&+\frac{\mathrm{i}}{2}\varphi\left(\arg(1-x)-\arg(-x)\right)\ln^2 x
\end{aligned}$$

(see Chapter 6 of [18]), which effectively enables one to get combinations of Li_3 of the only arguments R and $-R$.

Then one reduces the terms containing Li_2 as combinations of Li_2 of the only arguments R and $-R$. This is possible after using the Euler relations for Li_2, which read

$$\mathrm{Li}_2(z) + \mathrm{Li}_2(1-z) = \frac{\pi^2}{6} - \ln z\ln(1-z), \qquad (A.15)$$

and

$$\begin{aligned}
&\mathrm{Li}_2(z) + \mathrm{Li}_2\left(\frac{1}{z}\right) \qquad (A.16)\\
&= \frac{\pi^2}{3} - \frac{1}{2}\ln^2 z + \mathrm{i}\left(\arg(z-1)-\arg(1-z)\right)\ln z,
\end{aligned}$$

as well as the Landen relation

$$\begin{aligned}
&\mathrm{Li}_2(z) + \mathrm{Li}_2\left(\frac{-z}{1-z}\right) \\
&= -\frac{1}{2}\ln^2(1-z) \qquad (A.17)\\
&+\mathrm{i}\varphi\left(-\arg(1-z)\right)\left(\ln(1-z)-\ln z-\mathrm{i}\pi\right).
\end{aligned}$$

The most non-trivial transformation is based on the Hill formula (which enables one to expand the double variable function $\mathrm{Li}_2(x\,y)$) continuated in the whole complex plane (see [19] for details),

$$\begin{aligned}
\mathrm{Li}_2(x\,y) = \mathrm{Li}_2(x) + \mathrm{Li}_2(y) - \mathrm{Li}_2\left[\frac{x(1-y)}{1-xy}\right] \\
-\mathrm{Li}_2\left[\frac{y(1-x)}{1-xy}\right] - \ln\left[\frac{1-x}{1-xy}\right]\ln\left[\frac{1-y}{1-xy}\right] \\
+\mathrm{i}\varphi\left(\arg(1-y)-\arg(1-xy)\right)
\end{aligned}$$

$$
\begin{aligned}
&\times\left\{\ln\left[\frac{1-x}{1-xy}\right]-\ln\left[\frac{y(1-x)}{1-xy}\right]\right\}\\
&+\mathrm{i}\varphi\,(\arg(1-x)-\arg(1-xy))\\
&\times\left\{\ln\left[\frac{1-y}{1-xy}\right]-\ln\left[\frac{x(1-y)}{1-xy}\right]\right\}\\
&+\varphi\,(\arg(1-y)-\arg(1-xy))\\
&\times\varphi\,(\arg(1-x)-\arg(1-xy))\,. \qquad (A.18)
\end{aligned}
$$

After a long succession of painful simplifications, one finally gets

$$
\begin{aligned}
B=&-\frac{1}{64Q_1Q_2}\left(3R^2+2+\frac{3}{R^2}\right)\\
&\times\big(\ln(1-R)\ln^2 R-\ln(R+1)\ln^2 R\\
&-2\mathrm{Li}_2(-R)\ln R+2\mathrm{Li}_2(R)\ln R\\
&+2\mathrm{Li}_3(-R)-2\mathrm{Li}_3(R)\big)\,. \qquad (A.19)
\end{aligned}
$$

Using (A.8) and (A.19) in order to express $K=A+B$ and using (A.1), one proves the result (4.2).

The limits $R\gg 1$ and $R\ll 1$ can be easily extracted using the asymptotical formulae for a large argument of Li_2 and Li_3. In the case of Li_2 they read

$$
\begin{aligned}
\mathrm{Li}_2(x)\sim&-\frac{1}{2}\ln^2 x+\mathrm{i}\,(\arg(x)-\arg(-x))\ln x\\
&+\frac{\pi^2}{3}-\frac{1}{x}-\frac{1}{4x^2}-\frac{1}{9x^3}+o\left(\frac{1}{x^3}\right), \qquad (A.20)
\end{aligned}
$$

which reduces in the case of interest here to

$$
\begin{aligned}
\mathrm{Li}_2(x)\sim&-\frac{1}{2}\ln^2 x-\mathrm{i}\pi\ln x+\frac{\pi^2}{3}-\frac{1}{x}-\frac{1}{4x^2}-\frac{1}{9x^3}\\
&+o\left(\frac{1}{x^3}\right)\quad \text{for}\quad x\to+\infty, \qquad (A.21)\\
\mathrm{Li}_2(x)\sim&-\frac{1}{2}\ln^2 x+\mathrm{i}\pi\ln x+\frac{\pi^2}{3}-\frac{1}{x}-\frac{1}{4x^2}-\frac{1}{9x^3}\\
&+o\left(\frac{1}{x^3}\right)\quad \text{for}\quad x\to-\infty. \qquad (A.22)
\end{aligned}
$$

Similarly for Li_3 one gets from the functional relation (A.14) the asymptotic expansion

$$
\begin{aligned}
\mathrm{Li}_3(x)\sim&-\frac{1}{6}\ln^3(-x)-\frac{\pi^2}{6}\ln(-x)+\frac{1}{x}+\frac{1}{8x^2}\\
&+\frac{1}{27x^3}+o\left(\frac{1}{x^3}\right)\quad \text{for}\quad x\to\infty, \qquad (A.23)
\end{aligned}
$$

which reduces to

$$
\begin{aligned}
\mathrm{Li}_3(x)\sim&-\frac{1}{6}\ln^3 x-\mathrm{i}\frac{\pi}{2}\ln^2 x+\frac{\pi^2}{3}\ln x+\frac{1}{x} \qquad (A.24)\\
&+\frac{1}{8x^2}+\frac{1}{27x^3}+o\left(\frac{1}{x^3}\right)\quad \text{for}\quad x\to+\infty,
\end{aligned}
$$

and

$$
\begin{aligned}
\mathrm{Li}_3(x)\sim&-\frac{1}{6}\ln^3 x+\mathrm{i}\frac{\pi}{2}\ln^2 x+\frac{\pi^2}{3}\ln x+\frac{1}{x} \qquad (A.25)\\
&+\frac{1}{8x^2}+\frac{1}{27x^3}+o\left(\frac{1}{x^3}\right)\quad \text{for}\quad x\to-\infty.
\end{aligned}
$$

These asymptotical formulae immediately lead to

$$
\begin{aligned}
\mathcal{M}_{t_{\min}}\sim&+\mathrm{i}s\frac{N_c^2-1}{N_c^2}\alpha_s^2\alpha_{\mathrm{em}}\alpha(k_1)\beta(k_2)f_\rho^2\frac{96\pi^2}{Q_1^2Q_2^2}\\
&\times\left(\frac{\ln R}{R}-\frac{1}{6R}\right). \qquad (A.26)
\end{aligned}
$$

A.2 Computation of the amplitude at $t=t_{\min}$ in the partonic approach

In this appendix we rederive the asymptotic result (A.26) in a way which corresponds to the usual parton collinear limit. In that limit, this simple result agrees with the fact that each loop of t-channel gluons gives rise to at most one logarithmic term. The logarithmic term corresponds to the leading logarithm approximation (LLA), while the constant term goes beyond this approximation. Let us show that this result can be easily obtained from the representation (3.6). Indeed, in the partonic approach, $k_\perp^2$ is neglected with respect to any scale of the order of Q_1^2, and symmetrically any scale of the order of Q_2^2 is neglected with respect to $k_\perp^2$. It is clear that in such an approximation, one immediately recovers the dominant contribution $\frac{\ln R}{R}$ of (A.26). The question arises how to define a precise prescription in order to get the full leading twist expression (A.26). It turns out that the integral (A.2) provides the proper result

$$
K\sim\frac{1}{3Q_1Q_2}\left(\frac{\ln R}{R}-\frac{1}{6R}\right), \qquad (A.27)
$$

after expanding the integrand $\mathcal{K}$ at leading order, namely

$$
\mathcal{K}\sim\frac{1}{k^2Q_1^2z_1\bar{z}_1}, \qquad (A.28)
$$

and then integrating the result from $z_2\bar{z}_2/R$ to $Rz_1\bar{z}_1$. Let us justify this prescription. We rewrite the integral K of (A.2) as

$$
K=\frac{4}{Q_1Q_2}\int_0^{\frac{1}{2}}\mathrm{d}z_1\int_0^{\frac{1}{2}}\mathrm{d}z_2\int_0^{\infty}\mathrm{d}u\frac{z_1\bar{z}_1z_2\bar{z}_2}{(u+Rz_1\bar{z}_1)(u+z_2\bar{z}_2/R)}, \qquad (A.29)
$$

and separate the $u=k^2/(Q_1Q_2)$ integration as

$$
\int_0^{\infty}\mathrm{d}u=\int_{\beta Rz_1\bar{z}_1}^{\infty}\mathrm{d}u+\int_{\alpha\frac{z_2\bar{z}_2}{R}}^{\beta Rz_1\bar{z}_1}\mathrm{d}u+\int_0^{\alpha\frac{z_2\bar{z}_2}{R}}\mathrm{d}u, \qquad (A.30)
$$

where the parameters α and β are arbitrary. In the large R limit, $\beta Rz_1\bar{z}_1<\alpha\frac{z_2\bar{z}_2}{R}$ for $z_1<\frac{\alpha}{\beta}\frac{z_2\bar{z}_2}{R^2}$. Let us perform a systematic expansion of K in the limit where α and β satisfy $R \gg \alpha \gg 1$ and $R \gg 1/\beta \gg 1$. We organize the expansion in such a form that the large R limit is taken first (which means the dominant twist approximation), and only then the large α and small β limit are taken. Decompose $K = K_1 + K_2 + K_3 + K_4 + K_5 + K_6$, where

$$K_1 = \frac{4}{Q_1 Q_2} \times \int_0^{\frac{1}{2}} dz_2 \int_0^{\frac{\alpha}{\beta}\frac{z_2\bar{z}_2}{R^2}} dz_1 \int_{\alpha\frac{z_2\bar{z}_2}{R}}^{\infty} du \frac{z_1\bar{z}_1 z_2\bar{z}_2}{(u + Rz_1\bar{z}_1)(u + z_2\bar{z}_2/R)}, \tag{A.31}$$

$$K_2 = \frac{4}{Q_1 Q_2} \times \int_0^{\frac{1}{2}} dz_2 \int_0^{\frac{\alpha}{\beta}\frac{z_2\bar{z}_2}{R^2}} dz_1 \int_{\beta R z_1\bar{z}_1}^{\alpha\frac{z_2\bar{z}_2}{R}} du \frac{z_1\bar{z}_1 z_2\bar{z}_2}{(u + Rz_1\bar{z}_1)(u + z_2\bar{z}_2/R)}, \tag{A.32}$$

$$K_3 = \frac{4}{Q_1 Q_2} \times \int_0^{\frac{1}{2}} dz_2 \int_0^{\frac{\alpha}{\beta}\frac{z_2\bar{z}_2}{R^2}} dz_1 \int_0^{\beta R z_1\bar{z}_1} du \frac{z_1\bar{z}_1 z_2\bar{z}_2}{(u + Rz_1\bar{z}_1)(u + z_2\bar{z}_2/R)}, \tag{A.33}$$

$$K_4 = \frac{4}{Q_1 Q_2} \times \int_0^{\frac{1}{2}} dz_2 \int_{\frac{\alpha}{\beta}\frac{z_2\bar{z}_2}{R^2}}^{\frac{1}{2}} dz_1 \int_{\beta R z_1\bar{z}_1}^{\infty} du \frac{z_1\bar{z}_1 z_2\bar{z}_2}{(u + Rz_1\bar{z}_1)(u + z_2\bar{z}_2/R)}, \tag{A.34}$$

$$K_5 = \frac{4}{Q_1 Q_2} \times \int_0^{\frac{1}{2}} dz_2 \int_{\frac{\alpha}{\beta}\frac{z_2\bar{z}_2}{R^2}}^{\frac{1}{2}} dz_1 \int_{\alpha\frac{z_2\bar{z}_2}{R}}^{\beta R z_1\bar{z}_1} du \frac{z_1\bar{z}_1 z_2\bar{z}_2}{(u + Rz_1\bar{z}_1)(u + z_2\bar{z}_2/R)}, \tag{A.35}$$

$$K_6 = \frac{4}{Q_1 Q_2} \times \int_0^{\frac{1}{2}} dz_2 \int_{\frac{\alpha}{\beta}\frac{z_2\bar{z}_2}{R^2}}^{\frac{1}{2}} dz_1 \int_0^{\alpha\frac{z_2\bar{z}_2}{R}} du \frac{z_1\bar{z}_1 z_2\bar{z}_2}{(u + Rz_1\bar{z}_1)(u + z_2\bar{z}_2/R)}. \tag{A.36}$$

The integrals K_1, K_2 and K_3 are of order $1/R^3$ and can thus be neglected at leading twist. It corresponds to the absence of end-point singularities in the z variables and means that one could safely replace the lower bound of the z_1 integration in K_4, K_5 and K_6 by 0. Let us focus now on the integral K_5. The integration on u runs from $\alpha z_2\bar{z}_2/R$ to $\beta R z_1\bar{z}_1$. Since $z_1\bar{z}_1 \le 1/4$ and $z_2\bar{z}_2 \le 1/4$, in the limit $R \gg 1$, $\alpha \gg 1$ and $\beta \ll 1$, one can safely expand the integrand of K_5 in powers of $u/(Rz_1\bar{z}_1)$ and $z_2\bar{z}_2/(Ru)$. At leading twist, only the dominant term has to be kept, and $\mathcal{K}$ can be approximated by (A.28). Integration with respect to u, z_1 and z_2 then leads to

$$K_5 \sim \left(\frac{\ln R}{R} - \frac{1}{6R} + \frac{1}{3R}\ln\frac{\beta}{\alpha}\right). \tag{A.37}$$

The logarithmic contribution $\ln\frac{\beta}{\alpha}$ corresponds to the boundary effect and is to be completely compensated by K_4 and K_6, which behaves respectively as $1/R\ln\beta$ and $1/R\ln\alpha$ in the limit $R \gg 1$, $\alpha \gg 1$ and $\beta \ll 1$. This justifies the assumption stated at the beginning of this paragraph.

A.3 Integrals

In this appendix we collect all the generic integrals which appear in the computation of the Born amplitude. In addition to the integral I_2 defined in (4.7), we introduce the two following integrals with effective masses:

$$I_{2m} = \int \frac{d^d\underline{k}}{\underline{k}^2\left((\underline{k}-\underline{p})^2 + m^2\right)} \tag{A.38}$$

and

$$J_{2m_a m_b} = \int \frac{d^d\underline{k}}{\left((\underline{k}-\underline{a})^2 + m_a^2\right)\left((\underline{k}-\underline{b})^2 + m_b^2\right)}. \tag{A.39}$$

Let us first consider the case of the integral I_{2m}. Using the Feynman parametrization, one easily gets

$$I_{2m} = \pi^{1+\epsilon}\Gamma(1-\epsilon)(\underline{p}^2 + m^2)^{-1+\epsilon} \times \int_0^1 d\alpha\,\alpha^{\epsilon-1}\left(1 - \alpha\frac{\underline{p}^2}{\underline{p}^2 + m^2}\right)^{\epsilon-1}. \tag{A.40}$$

In the massless case, one immediately obtains

$$I_2 = \frac{2\pi}{\underline{p}^2\epsilon}\left(1 + \epsilon\left(\ln(\pi\underline{p}^2) - \Psi(1)\right)\right). \tag{A.41}$$

For the non-zero mass case, the α integration leads to the hypergeometric function ${}_2\mathrm{F}_1$,

$$I_{2m} = \pi^{1+\epsilon}\frac{\Gamma(1-\epsilon)}{\epsilon(\underline{p}^2 + m^2)^{1-\epsilon}}\,{}_2F_1\left(1-\epsilon, \epsilon, 1+\epsilon; \frac{\underline{p}^2}{\underline{p}^2 + m^2}\right). \tag{A.42}$$

After performing the Euler transformation $z \to z/(z-1)$ in the argument of the hypergeometric function and then expanding the result in powers of ϵ, one gets

$$I_{2m} = \frac{\pi}{\epsilon(\underline{p}^2 + m^2)} \times \left(1 + \epsilon\left(\ln\pi - \Psi(1) - \ln m^2 + 2\ln(\underline{p}^2 + m^2)\right)\right). \tag{A.43}$$

Let us now turn to the more general case where the propagators contain two (different) masses. In this case dimensional regularization is not necessary, and after straightforward calculations, using the Feynman parametrization, one obtains

$$J_{2m_am_b} = \pi \Bigg/ \left\{ \left[\left((\underline{a}-\underline{b})^2 + (m_a - m_b)^2 \right) \times \left((\underline{a}-\underline{b})^2 + (m_a + m_b)^2 \right) \right]^{1/2} \right\} \tag{A.44}$$
$$\times \ln \left| \left\{ (\underline{a}-\underline{b})^2 + m_a^2 + m_b^2 + \left[\left((\underline{a}-\underline{b})^2 + (m_a - m_b)^2 \right) \times \left((\underline{a}-\underline{b})^2 + (m_a + m_b)^2 \right) \right]^{1/2} \right\} \Bigg/ \left\{ (\underline{a}-\underline{b})^2 + m_a^2 + m_b^2 - \left[\left((\underline{a}-\underline{b})^2 + (m_a - m_b)^2 \right) \times \left((\underline{a}-\underline{b})^2 + (m_a + m_b)^2 \right) \right]^{1/2} \right\} \right| .$$

For the purpose of our computation, we will need the previous integral only for the special case where $\underline{a}$ and $\underline{b}$ are collinear.

In that case, one obtains, with $r = |\underline{r}|$,

$$J_{2\alpha\beta} = \int \frac{\mathrm{d}^d \underline{k}}{\left((\underline{k} - \underline{r}a)^2 + \alpha^2 \right) \left((\underline{k} - \underline{r}b)^2 + \beta^2 \right)} = \frac{\pi}{\sqrt{\lambda}} \ln \frac{r^2(a-b)^2 + \alpha^2 + \beta^2 + \sqrt{\lambda}}{r^2(a-b)^2 + \alpha^2 + \beta^2 - \sqrt{\lambda}},$$

where we introduce the notation

$$\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz, \tag{A.45}$$

which enables us to define, for the purpose of our computation,

$$\lambda = \lambda(-r^2(a-b)^2, \alpha^2, \beta^2) = (\alpha^2 - \beta^2)^2 + 2(\alpha^2 + \beta^2) r^2 (a-b)^2 + r^4 (a-b)^4. \tag{A.46}$$

Let us first consider the integral I_{3m}. We start from the following identity:

$$\int \frac{\mathrm{d}^2 \underline{k}}{\underline{k}^2 (\underline{k}-\underline{p})^2} \times \left(\frac{1}{(\underline{k}-\underline{a})^2 + m^2} - \frac{1}{\underline{a}^2 + m^2} - \frac{1}{(\underline{p}-\underline{a})^2 + m^2} + \frac{1}{(\underline{k}-\underline{p}+\underline{a})^2 + m^2} \right)$$
$$= -\left(\frac{1}{\underline{a}^2 + m^2} + \frac{1}{(\underline{p}-\underline{a})^2 + m^2} \right) \int \frac{\mathrm{d}^d \underline{k}}{\underline{k}^2 (\underline{k}-\underline{p})^2} + \int \frac{\mathrm{d}^d \underline{k}}{\underline{k}^2 (\underline{k}-\underline{p})^2 \left((\underline{k}-\underline{a})^2 + m^2 \right)} + \int \frac{\mathrm{d}^d \underline{k}}{\underline{k}^2 (\underline{k}-\underline{p})^2 \left((\underline{k}-\underline{p}+\underline{a})^2 + m^2 \right)}. \tag{A.47}$$

This identity relates a finite expression on the LHS with a sum of dimensionally regularized integrals. After shifting the integration variable in the last integral on the RHS of (A.47), one obtains

$$\int \frac{\mathrm{d}^d \underline{k}}{\underline{k}^2 (\underline{k}-\underline{p})^2 \left((\underline{k}-\underline{a})^2 + m^2 \right)} \tag{A.48}$$
$$= \frac{1}{2} \left(\frac{1}{\underline{a}^2 + m^2} + \frac{1}{(\underline{p}-\underline{a})^2 + m^2} \right) \int \frac{\mathrm{d}^d \underline{k}}{\underline{k}^2 (\underline{k}-\underline{p})^2} + \frac{1}{2} \int \frac{\mathrm{d}^2 \underline{k}}{\underline{k}^2 (\underline{k}-\underline{p})^2} \times \left(\frac{1}{(\underline{k}-\underline{a})^2 + m^2} - \frac{1}{\underline{a}^2 + m^2} - \frac{1}{(\underline{p}-\underline{a})^2 + m^2} + \frac{1}{(\underline{k}-\underline{p}+\underline{a})^2 + m^2} \right),$$

which expresses I_{3m} in terms of the divergent integral I_2 already calculated and a finite integral whose computation is our next task. The method which we use is the generalization for the massive case, in momentum space, of the technique of the calculation of massless two dimensional diagrams in the coordinate space encountered in conformal field theories [20].

The essential point is to perform in two dimensional finite integrals a conformal transformation of the type $\underline{l} \to \underline{l}/\underline{l}^2$ on the integration variables and vector parameters, and of the type $m^2 \to 1/m^2$ for the dimensionful parameters. This transformation reduces the number of propagators.

Let us illustrate this method in the special case where $\underline{a}$ and $\underline{p}$ are collinear, which is of practical interest for our computation. We will thus focus on the finite integral in the RHS of (A.48), which turns out to be J_{3m}, as defined in (4.11). The transformation

$$\underline{k} \to \frac{\underline{K}}{\underline{K}^2}, \quad \underline{r} \to \frac{\underline{R}}{\underline{R}^2}, \quad m \to \frac{1}{M} \tag{A.49}$$

reduces the number of propagators and gives

$$J_{3m} = \int \frac{\mathrm{d}^2 \underline{k}}{\underline{k}^2 (\underline{k}-\underline{r})^2} \times \left(\frac{1}{(\underline{k}-\underline{r}a)^2 + m^2} - \frac{1}{\underline{r}^2 + m^2} + (a \leftrightarrow \bar{a}) \right)$$
$$= R^2 \int \frac{\mathrm{d}^2 \underline{K}}{(\underline{K}-\underline{R})^2} \times \left(\frac{K^2 R^2}{(\underline{R} - a\underline{K})^2 + \frac{\underline{K}^2 \underline{R}^2}{M^2}} - \frac{1}{a^2 r^2 + m^2} + (a \leftrightarrow \bar{a}) \right). \tag{A.50}$$

After performing the shift of variable $\underline{K} = \underline{R} + \underline{k}'$ and then finally making the inverse transformation

$$\underline{k}' \to \frac{\underline{k}}{\underline{k}^2}, \quad \underline{R} \to \frac{\underline{r}}{\underline{r}^2}, \quad M \to \frac{1}{m}, \tag{A.51}$$

we end up with

$$J_{3m} = \frac{1}{r^2} \int \frac{\mathrm{d}^2\underline{k}}{\underline{k}^2} \times \left[\frac{(\underline{r}+\underline{k})^2}{(r^2a^2+m^2)} \times \frac{1}{\left(\left(\underline{k} - \underline{r}\frac{r^2 a\underline{a} - m^2}{r^2\underline{a}^2+m^2}\right)^2 + \frac{m^2r^4}{(r^2\underline{a}^2+m^2)^2}\right)} - \frac{1}{a^2r^2+m^2} + (a \leftrightarrow \bar{a}) \right]. \tag{A.52}$$

The computation of this integral can now be performed using the standard Feynman parameter technique. It results in

$$J_{3m} = \frac{2\pi}{r^2}\left\{\left(\frac{1}{r^2a^2+m^2} - \frac{1}{r^2\bar{a}^2+m^2}\right)\ln\frac{r^2a^2+m^2}{r^2\bar{a}^2+m^2} + \left(\frac{1}{r^2a^2+m^2} + \frac{1}{r^2\bar{a}^2+m^2} + \frac{2}{r^2a\bar{a}-m^2}\right) \times \ln\frac{(r^2a^2+m^2)(r^2\bar{a}^2+m^2)}{m^2r^2}\right\}. \tag{A.53}$$

Let us now focus on the I_{4mm} integral. We start with an identity analogous as those in (A.47), which leads to

$$\int \frac{\mathrm{d}^d\underline{k}}{\underline{k}^2(\underline{k}-\underline{p})^2\left((\underline{k}-\underline{a})^2+m_a^2\right)(\underline{k}-\underline{b})^2+m_b^2)} \tag{A.54}$$
$$= \frac{1}{2}\left(\frac{1}{(\underline{a}^2+m_a^2)(\underline{b}^2+m_b^2)} + \frac{1}{\left((\underline{p}-\underline{a})^2+m_a^2\right)\left((\underline{p}-\underline{b})^2+m_b^2\right)}\right)\int\frac{\mathrm{d}^d\underline{k}}{\underline{k}^2(\underline{k}-\underline{p})^2} + \frac{1}{2}J,$$

where

$$J = \int \frac{\mathrm{d}^2\underline{k}}{\underline{k}^2(\underline{k}-\underline{p})^2} \times \left[\frac{1}{\left((\underline{k}-\underline{a})^2+m_a^2\right)\left((\underline{k}-\underline{b})^2+m_b^2\right)} - \frac{1}{(\underline{a}^2+m_a^2)(\underline{b}^2+m_b^2)} - \frac{1}{\left((\underline{p}-\underline{a})^2+m_a^2\right)\left((\underline{p}-\underline{b})^2+m_b^2\right)} + \frac{1}{\left((\underline{k}-\underline{p}+\underline{a})^2+m_a^2\right)\left((\underline{k}-\underline{p}+\underline{b})^2+m_b^2\right)}\right]. \tag{A.55}$$

This identity expresses the integral $I_{4m_am_b}$ as a sum of an IR divergent integral, which has already been computed in (A.41), and of a finite integral J, on which we will apply the same trick based on conformal transformations. Once more, although our method can be applied to the previous integral J, we will restrict ourselves to the more simple case where a and b are collinear. It thus means that we need to compute the integral $J_{4\mu_1\mu_2}$ as defined in (4.12). We apply on $J_{4\mu_1\mu_2}$ three successive transformations accompanied by appropriate changes of variables as described above: a conformal transformation, a shift of integration variable and an inverse conformal transformation. This gives

$$J_{4\mu_1\mu_2} = \frac{1}{\underline{r}^2}\int\frac{\mathrm{d}^d\underline{k}}{\underline{k}^2} \times \left[\frac{\left((\underline{k}-\underline{r})^2\right)^2}{(r^2\bar{z}_1+\mu_1^2)} \times \frac{1}{\left(\left(\underline{k}+\underline{r}\frac{z_1\bar{z}_1r^2-\mu_1^2}{\bar{z}_1^2r^2+\mu_1^2}\right)^2+\frac{r^4\mu_1^2}{(r^2z_1^2+\mu_1^2)^2}\right)} \times \frac{1}{(r^2\bar{z}_2+\mu_2^2)} \times \frac{1}{\left(\left(\underline{k}+\underline{r}\frac{z_2\bar{z}_2r^2-\mu_2^2}{\bar{z}_2^2r^2+\mu_2^2}\right)^2+\frac{r^4\mu_2^2}{(r^2z_2^2+\mu_2^2)^2}\right)} - \frac{1}{(r^2\bar{z}_1+\mu_1^2)(r^2\bar{z}_2+\mu_2^2)} + (z \leftrightarrow \bar{z})\right]. \tag{A.56}$$

In this expression, the integral which remains to be computed has the form

$$J_{3\alpha\beta} = \int\frac{\mathrm{d}^2\underline{k}\left((\underline{k}-\underline{r})^2\right)^2}{\underline{k}^2[(\underline{k}-\underline{r}a)^2+\alpha^2]\,[(\underline{k}-\underline{r}b)^2+\beta^2]}. \tag{A.57}$$

This integral can be rewritten in the form

$$J_{3\alpha\beta} = J_{3\alpha\beta}^{\mathrm{UV}} + J_{3\alpha\beta}^{\mathrm{IR}}, \tag{A.58}$$

where

$$J_{3\alpha\beta}^{\mathrm{UV}} = \int\frac{\mathrm{d}^d\underline{k}\;\underline{k}^2}{[(\underline{k}-\underline{r}a)^2+\alpha^2]\,[(\underline{k}-\underline{r}b)^2+\beta^2]} \tag{A.59}$$

and

$$J^{\rm IR}_{3\alpha\beta} = \int \frac{{\rm d}^d \underline{k}\left[\left((\underline{k}-\underline{r})^2\right)^2 - (\underline{k}^2)^2\right]}{\underline{k}^2[(\underline{k}-\underline{r}a)^2+\alpha^2]\,[(\underline{k}-\underline{r}b)^2+\beta^2]}. \quad \text{(A.60)}$$

$J^{\rm UV}_{3\alpha\beta}$ is IR finite but UV divergent. On the contrary $J^{\rm IR}_{3\alpha\beta}$ is UV finite but IR divergent. The calculation of $J^{\rm UV}_{3\alpha\beta}$ is standard, although somewhat lengthy, and leads to the result

$$\begin{aligned} J^{\rm UV}_{3\alpha\beta} = \pi\Bigg\{ & -\frac{\pi^\epsilon\Gamma(1-\epsilon)}{\epsilon} - \frac{1}{2}\ln(\alpha^2\beta^2) \\ & + \frac{\alpha^2-\beta^2}{2r^2(a-b)^2}\ln\frac{\alpha^2}{\beta^2} \\ & - \frac{\sqrt{\lambda}}{2r^2(a-b)^2}\ln\frac{r^2(a-b)^2+\alpha^2+\beta^2+\sqrt{\lambda}}{r^2(a-b)^2+\alpha^2+\beta^2-\sqrt{\lambda}} \\ & - \frac{[\sqrt{\lambda}-\alpha^2+\beta^2-r^2(a^2-b^2)]^2}{4r^2(a-b)^2\sqrt{\lambda}} \\ & \times \ln\frac{\sqrt{\lambda}-r^2(a-b)^2-\alpha^2+\beta^2}{\sqrt{\lambda}+r^2(a-b)^2-\alpha^2+\beta^2} \\ & - \frac{[\sqrt{\lambda}-\beta^2+\alpha^2-r^2(b^2-a^2)]^2}{4r^2(a-b)^2\sqrt{\lambda}} \\ & \times \ln\frac{\sqrt{\lambda}-r^2(a-b)^2+\alpha^2-\beta^2}{\sqrt{\lambda}+r^2(a-b)^2+\alpha^2-\beta^2}\Bigg\}. \quad \text{(A.61)} \end{aligned}$$

The evaluation of $J^{\rm IR}_{3\alpha\beta}$ can be simplified by expressing it in the following way:

$$\begin{aligned} J^{\rm IR}_{3\alpha\beta} = & \int {\rm d}^d\underline{k}\frac{2\underline{r}^2}{[(\underline{k}-\underline{r}a)^2+\alpha^2]\,[(\underline{k}-\underline{r}b)^2+\beta^2]} \\ & + r^4\int\frac{{\rm d}^d\underline{k}}{\underline{k}^2[(\underline{k}-\underline{r}a)^2+\alpha^2][(\underline{k}-\underline{r}b)^2+\beta^2]} \\ & - 4r^2\int\frac{{\rm d}^d\underline{k}\;\underline{k}\cdot\underline{r}}{\underline{k}^2[(\underline{k}-\underline{r}a)^2+\alpha^2][(\underline{k}-\underline{r}b)^2+\beta^2]} \\ & - 4\int\frac{{\rm d}^d\underline{k}\;\underline{k}\cdot\underline{r}}{[(\underline{k}-\underline{r}a)^2+\alpha^2]\,[(\underline{k}-\underline{r}b)^2+\beta^2]} \quad \text{(A.62)} \\ & + 4\int\frac{{\rm d}^d\underline{k}(\underline{k}\cdot\underline{r})^2}{\underline{k}^2[(\underline{k}-\underline{r}a)^2+\alpha^2]\,[(\underline{k}-\underline{r}b)^2+\beta^2]}. \end{aligned}$$

Each of these integrals is regular, except the second one, which diverges like

$$J^{\rm IR\,div.}_{3\alpha\beta} \equiv \frac{\pi^{(1+\epsilon)}}{\epsilon\Gamma(1+\epsilon)}\frac{r^4}{(r^2a^2+\alpha^2)(r^2b^2+\beta^2)}. \quad \text{(A.63)}$$

The evaluation of each of these integrals is lengthy but straighforward after using the Feynman parametrization.

Since IR and UV divergences cancel when evaluating the integral defined in (A.56), we effectively need the finite part of $J_{3\alpha\beta}$, defined as the remaining term after removing the pole in $1/\epsilon$, which sums up the finite part of $J^{\rm IR}_{3\alpha\beta}$ and of $J^{\rm UV}_{3\alpha\beta}$. With this definition, one can express $J_{4\mu_1\mu_2}$ as

$$\begin{aligned} J_{4\mu_1\mu_2} = & \frac{1}{r^2(r^2\bar{z}_1^2+\mu_1^2)(r^2\bar{z}_2^2+\mu_2^2)} \quad \text{(A.64)} \\ & \times J^{\rm finite}_{3\alpha\beta}\left(\frac{-z_1\bar{z}_1r^2+\mu_1^2}{\bar{z}_1^2r^2+\mu_1^2}, \frac{-z_2\bar{z}_2r^2+\mu_2^2}{\bar{z}_2^2r^2+\mu_2^2},\right. \\ & \left.\frac{r^2\mu_1}{r^2\bar{z}_1^2+\mu_1^2}, \frac{r^2\mu_2}{r^2\bar{z}_2^2+\mu_2^2}, r\right) \\ & + \frac{1}{r^2(r^2z_1^2+\mu_1^2)(r^2z_2^2+\mu_2^2)} \\ & \times J^{\rm finite}_{3\alpha\beta}\left(\frac{-z_1\bar{z}_1r^2+\mu_1^2}{z_1^2r^2+\mu_1^2}, \frac{-z_2\bar{z}_2r^2+\mu_2^2}{z_2^2r^2+\mu_2^2},\right. \\ & \left.\frac{r^2\mu_1}{r^2z_1^2+\mu_1^2}, \frac{r^2\mu_2}{r^2z_2^2+\mu_2^2}, r\right), \end{aligned}$$

with

$$\begin{aligned} & J^{\rm finite}_{3\alpha\beta}(a,\,b,\alpha,\beta,r) \quad \text{(A.65)} \\ & = \pi\Bigg\{\frac{1}{2\sqrt{\lambda}}\Bigg[-\frac{(\alpha-\beta)^2(\alpha+\beta)^2}{(a-b)^2r^2} - 4\frac{\alpha^2-\beta^2}{a-b} \\ & -2(\alpha^2+\beta^2) - \left((a-b)^2+4(a+b)-12\right)r^2 \\ & -r^2\left(\frac{1}{r^2a^2+\alpha^2}+\frac{1}{r^2b^2+\beta^2}\right)\Bigg] \\ & \times\ln\frac{r^2(a-b)^2+\alpha^2+\beta^2+\sqrt{\lambda}}{r^2(a-b)^2+\alpha^2+\beta^2-\sqrt{\lambda}} \\ & + \frac{1}{\sqrt{\lambda}}\Bigg[r^2(a-b)^2+2(\alpha^2+\beta^2)-2\frac{a\beta^2-b\alpha^2}{a-b} \\ & +2abr^2+r^2\frac{\alpha^2-\beta^2+(a^2-b^2)r^2}{b(r^2a^2+\alpha^2)-a(r^2b^2+\beta^2)} \\ & \times\left(\frac{r^2a}{r^2a^2+\alpha^2}+\frac{r^2b}{r^2b^2+\beta^2}-4\right) \\ & + \frac{(\alpha^2-\beta^2)^2+2(\alpha^2+\beta^2)^2r^2(a-b)^2+r^4(a-b)^4}{b(r^2a^2+\alpha^2)-a(r^2b^2+\beta^2)} \\ & \times\frac{2}{a-b}+\frac{(\alpha^2-\beta^2)^2}{r^2(a-b)^2}\Bigg] \\ & \times\ln\frac{\sqrt{\lambda}+r^2(a-b)^2+\alpha^2-\beta^2}{\sqrt{\lambda}-r^2(a-b)^2+\alpha^2-\beta^2} \\ & + \Bigg[-\frac{1}{ab(a-b)}\frac{a^2\beta^2-b^2\alpha^2+ab\sqrt{\lambda}}{a(r^2b^2+\beta^2)-b(r^2a^2+\alpha^2)} \\ & -\frac{r^2}{2\sqrt{\lambda}}\left(\frac{r^2a}{r^2a^2+\alpha^2}+\frac{r^2b}{r^2b^2+\beta^2}-4\right) \\ & \times\frac{\sqrt{\lambda}+\alpha^2-\beta^2-r^2(b^2-a^2)}{a(r^2b^2+\beta^2)-b(r^2a^2+\alpha^2)} \end{aligned}$$

$$
\begin{aligned}
&+\frac{2}{a-b}+\frac{a\alpha^2-b\beta^2}{(a-b)\sqrt{\lambda}} \\
&+\frac{(\alpha^2-\beta^2)^2}{2r^2(a-b)^2\sqrt{\lambda}}+\frac{(a^2+b^2)r^2}{2\sqrt{\lambda}}-\frac{a+b}{2(a-b)}\Bigg]\ln\frac{\alpha^2}{\beta^2} \\
&+\left[\frac{1}{ab}-\frac{r^4}{2(r^2a^2+\alpha^2)(r^2b^2+\beta^2)}-\frac{1}{2}\right]\ln\frac{\alpha^2\beta^2}{r^4} \\
&+\frac{1}{a(r^2b^2+\beta^2)-b(r^2a^2+\alpha^2)} \\
&\times\left[r^2\left(\frac{r^2a}{r^2a^2+\alpha^2}+\frac{r^2b}{r^2b^2+\beta^2}-4\right)+\frac{a^2r^2+\alpha^2}{a}\right. \\
&\left.+\frac{b^2r^2+\beta^2}{b}\right]\ln\frac{r^2a^2+\alpha^2}{r^2b^2+\beta^2} \\
&-\frac{(r^2a^2+\alpha^2)(r^2b^2+\beta^2)-abr^4}{ab(r^2a^2+\alpha^2)(r^2b^2+\beta^2)} \\
&\times\ln\frac{(r^2a^2+\alpha^2)(r^2b^2+\beta^2)}{r^4}\Bigg\},
\end{aligned}
$$

where appropriate additional $\ln r^2$ terms have been introduced after extracting the finite part of $J^{\rm UV}_{3\alpha\beta}$ and $J^{\rm IR}_{3\alpha\beta}$ in order to write the final result by logarithms of dimensionless quantities. This is possible since the final result $J_{4\mu_1\mu_2}$ is UV and IR finite.

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